

Lectures on Cartan differential geometry

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Lun 14-06-21 16:59

À : [_List_DL_CentreAGIF <_List_DL_CentreAGIF@umons.ac.be>](mailto:_List_DL_CentreAGIF@umons.ac.be)

Dear Colleagues,

For your information, with my collaborator Zhenya Skvortsov in the unit, together with Xavier Bekaert (U. Tours) and Maxim Grigoriev (Lomonosov state U. Moscow), we are co-organising a series of pedagogical lectures on Cartan differential geometry.

The lectures will be delivered by my collaborator Jordan François from the unit (whom you know well) and a colleague from the group of Joel Fine at ULB, Yannick Herfray.

The lectures will be broadcasted via Zoom. See the links and the program below.

Best regards
Nicolas Boulanger

Begin forwarded message:

From: Xavier Bekaert <Xavier.Bekaert@lmpt.univ-tours.fr>

Subject: Warm-up lectures before ESI program

Date: 14 June 2021 at 16:42:50 CEST

To: undisclosed-recipients;;

Dear colleagues,

The series of short lectures by Jordan François (UMONS) and Yannick Herfray (ULB) will start this week. They will be held in hybrid format (physically in Mons and online via Zoom).

Please find the Zoom links (together with the program and schedule) of these informal introductory lectures on conformal geometry à la Cartan (and related subjects) intended for non-experts as a self-contained warm-up for the ESI program.

The planned schedule is (Brussels timezone):

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Thursday 17/06: 14:00 - 16:00 (Jordan)

<https://us02web.zoom.us/j/85483216247?pwd=ZCt2YWE3ZkZqYmh3U2o2dUZsMHRQT09>

Code: 245867

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Friday 18/06: 10:00 - 12:00 (Jordan)

<https://us02web.zoom.us/j/81540150557?pwd=amk0Tm1ndjlxQ2N1TmFEUTFPVlhaQT09>

Code: 619888

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Monday 21/06: 11:00 - 12:00 (Jordan)

<https://us02web.zoom.us/j/85402541108?pwd=TGJiZTlxTTRhL1poeVFYVky5T2NlZz09>

Code: 803557

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Monday 21/06: 14:00 - 15:00 (Yannick)

<https://us02web.zoom.us/j/89184926414?pwd=USs4THNEazc1U05SS2Ftd1dQaXdrQT09>

Code: 884366

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Tuesday 22/06: 14:00 - 16:00 (Yannick)

<https://us02web.zoom.us/j/81352037647?pwd=ODdXd1dOaUxRYUxiazhsd21Nd2pnUT09>

Code: 054780

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Thursday 24/06: 14:00 - 16:00 (Yannick)

<https://us02web.zoom.us/j/85692934577?pwd=dUZHAmFJeTk0TUZpWlQ3UHp5cE9vdz09>

Code: 536933

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You are welcome to advertise this information to the other people in your local institution, but the idea is to keep the audience small in such a way that the atmosphere remains informal and people can easily ask questions.

People interested should contact <xavier.bekaert@lmpt.univ-tours.fr> to be included in the mailing list.

May the higher-spin force be with you,
Maxim, Nicolas, Xavier, Zhenya

PS: Tentative plan of the lectures

Jordan (5h)

* Reminder on principal bundles (1h)

- Tangent & vertical bundles, remarkable space of forms (equivariant, tensorial, basic)
- Bundle automorphism and gauge transformations
- Associated bundles (to a principal bundle)
- local structure and physics
- Non canonical structure: connections -> reminder on def Ehresmann connections

* Cartan geometry (2h30 or 3h)

- Cartan connection def -> Cartan geom & gauge gravity.
- TM as an associated bundle to a Cartan geometry (soldering) - flat

Cartan = Klein

- Ex of Cartan geom: reductive (Cartan-Riemann, Cartan-deSitter, CS action, McDowell-Mansouri?), parabolic (Cartan conformal, projective)
- Cartan connections as Ehresmann connections on a "bigger" principal bundle.
- Normal Cartan connection: the idea, characterisation via Spencer operator, ex in Cartan-Riemann & Cartan conformal.

* Conformal Cartan geometry & conformal gravity + tractor/twistors (1h30 or 1h)

- YM lagrangian and YM equation for normal conf Cartan (& twistor) connection & equiv to Weyl Lagrangian and Bach equation.
- Conformal Tractor & twistor bundles: 3 approaches (1 and 3 just mentioned for refs)
- > bottom-up (local) by prolongation of diff eq: Gover-et-al (tractors), Penrose-Rindler (twistors).
- > as associated to conf Cartan geom (global)

via Weyl structures: Cap-Slovak. ~ Yannick's presentation

- > top-down (local) via dressing: user friendly, details and refs only on request.

Yannick (5h)

* The tractor bundle from the perspective of FG ambient metric (2h = detailed presentation)

(all point below will be illustrated by a detailed presentation of the flat case in terms of the flat ambient metric)

- the bundle of scale
 - the canonical $ISO(p,q) \times \mathbb{R}$ principal bundle of Conformal geometry - the tractor bundle (filtration, metric)
 - Thomas operator and splitting isomorphisms
 - Cartan normal connection
 - FG ambient metric
- * Tractors and Einstein's equations (1h = detailed presentation)
- the flat model (Infinity tractor)
 - Einstein's equations (prop, reduction to Mcdowell-Mansouri) - Tractor Lagrangian (lightning presentation)
- * Tractors and asymptotically simple space-times (1h = overview)
- flat model: AdS/Minkowski/dS
 - Asymptotically simple/flat space-times (classical and "tractor" definition)
 - The induced tractor bundle on conformal infinity
 - The induced Cartan connection on conformal infinity
- * Local twistors and introduction to twistor theory (1h = overview)
- local twistors
 - the twistor correspondence: the flat model
 - Penrose/ Ward transform (lightning review)
 - Curved twistor theory: local twistors vs googly problem