DIFFERENTIAL ALGEBRA, ORDERED FIELDS AND MODEL THEORY

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This thesis deals with several issues concerning differential fields endowed with an order. It considers a particular theory, whose models are called "closed ordered differential fields". That theory was introduced in 1978 by Singer who used the acronym CODF to name it.

We undertake the task of describing differential polynomials vanishing or having positive values on a given set of points of a model of CODF. This leads to differential analogs of the nullstellensatz and the positivstellensatz for real closed fields, respectively proved by Dubois, Krivine and Risler, and Stengle. We present two differential versions of the positivstellensatz, a topological one and a purely algebraic one.

Linear differential Galois theory was generalised by Kolchin via strongly normal extensions. For a strongly normal extension of formally real fields whose field of constants is real closed, we study the differential Galois group and the relation between its definable subgroups and the intermediate extensions. This work conveys a sense of continuity of the approach taken by model theorists to investigate Galois theory. Elimination of quantifier and elimination of imaginaries play a central role in its development.

Eventually, we consider questions on the model theory of closed ordered differential fields. The main result we obtain is a characterisation of definable types that implies the density of definable types in the Stone space of CODF.